Identità ed Espressioni Goniometriche

Esercizi svolti sulle espressioni e identità goniometriche, che coinvolgono tutte le formule: archi associati, addizione e sottrazione, duplicazione, bisezione, Werner , prostaferesi.

Le tracce degli esercizi svolti sono state prese dal Libro :

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Titolo del libro: Matematica Blu 2.0

Casa Editrice : Zanichelli

Formulario di goniometria

Funzioni goniometriche di angoli particolari

Gradi	Radianti	Seno	Coseno	Tangente	Cotangente
$0°$	0	0	1	0	*non esiste*
$15°$	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$18°$	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
$22°30'$	$\frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$36°$	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$54°$	$\frac{3}{10}\pi$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$	$\sqrt{5-2\sqrt{5}}$
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$67°30'$	$\frac{3}{8}\pi$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
$72°$	$\frac{2}{5}\pi$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
$75°$	$\frac{5}{12}\pi$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$90°$	$\frac{\pi}{2}$	1	0	*non esiste*	0
$180°$	π	0	-1	0	*non esiste*
$270°$	$\frac{3}{2}\pi$	-1	0	*non esiste*	0
$360°$	2π	0	1	0	*non esiste*

Relazioni fondamentali tra le funzioni goniometriche di uno stesso angolo

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} \qquad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha}$$

Funzioni goniometriche di angoli associati

$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos\alpha$	$\tan(-\alpha) = -\tan\alpha$	$\cot(-\alpha) = -\cot\alpha$
$\sin(2\pi-\alpha) = -\sin\alpha$	$\cos(2\pi-\alpha) = \cos\alpha$	$\tan(2\pi-\alpha) = -\tan\alpha$	$\cot(2\pi-\alpha) = -\cot\alpha$
$\sin(\pi-\alpha) = \sin\alpha$	$\cos(\pi-\alpha) = -\cos\alpha$	$\tan(\pi-\alpha) = -\tan\alpha$	$\cot(\pi-\alpha) = -\cot\alpha$
$\sin(\pi+\alpha) = -\sin\alpha$	$\cos(\pi+\alpha) = -\cos\alpha$	$\tan(\pi+\alpha) = \tan\alpha$	$\cot(\pi+\alpha) = \cot\alpha$
$\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$	$\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$	$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$	$\cot\left(\frac{\pi}{2}-\alpha\right) = \tan\alpha$
$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$	$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$	$\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$	$\cot\left(\frac{\pi}{2}+\alpha\right) = -\tan\alpha$

Formule di addizione e sottrazione

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\begin{cases} \tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \\ \alpha+\beta, \alpha, \beta \neq \dfrac{\pi}{2} + k\pi \end{cases}$$

$$\begin{cases} \tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \\ \alpha-\beta, \alpha, \beta \neq \dfrac{\pi}{2} + k\pi \end{cases}$$

Formule di duplicazione

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\begin{cases} \tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} \\ \alpha \neq \dfrac{\pi}{4} + k\dfrac{\pi}{2} \wedge \alpha \neq \dfrac{\pi}{2} + k\pi \end{cases}$$

Formule di bisezione

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \qquad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \quad (\alpha \neq \pi + 2k\pi)$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} \quad \text{con } \alpha \neq \pi + 2k\pi$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} \quad \text{con } \alpha \neq k\pi$$

Formule parametriche

$$\sin\alpha = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-t^2}{1+t^2} \qquad \left(t = \tan\frac{\alpha}{2}, \quad \alpha \neq \pi + 2k\pi\right)$$

Formule di prostaferesi

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2} \qquad \cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2} \qquad \cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

Formule di Werner

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

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$$\cos\left(\alpha - \frac{2}{3}\pi\right) + \frac{1}{2}\operatorname{sen}\alpha + \cos\left(\alpha - \frac{7}{6}\pi\right) - \operatorname{sen}\left(\alpha - \frac{\pi}{6}\right) =$$

utilizziamo le formule di addizione e sottrazione

$$= \cos\alpha \cdot \cos\left(\frac{2}{3}\pi\right) + \operatorname{sen}\alpha \cdot \operatorname{sen}\left(\frac{2}{3}\pi\right) + \frac{1}{2}\operatorname{sen}\alpha +$$

$$+ \cos\alpha \cdot \cos\frac{7}{6}\pi + \operatorname{sen}\alpha \cdot \operatorname{sen}\frac{7}{6}\pi - \left[\operatorname{sen}\alpha \cdot \cos\left(\frac{\pi}{6}\right) -\right.$$

$$\left. - \cos\alpha \cdot \operatorname{sen}\left(\frac{\pi}{6}\right)\right] =$$

Dalle tabelle dei valori sappiamo che

$$\cos\frac{2}{3}\pi = -\frac{1}{2} \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\operatorname{sen}\frac{2}{3}\pi = \frac{\sqrt{3}}{2} \qquad \operatorname{sen}\frac{\pi}{6} = \frac{1}{2}$$

$$\cos\frac{7}{6}\pi = -\frac{\sqrt{3}}{2}$$

$$\operatorname{sen}\frac{7}{6}\pi = -\frac{1}{2}$$

da cui

$$= -\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2} \cdot \operatorname{sen}\alpha + \frac{1}{2}\operatorname{sen}\alpha + \left(-\frac{\sqrt{3}}{2}\right) \cdot \cos\alpha +$$

$$+ \left(-\frac{1}{2}\right) \cdot \operatorname{sen}\alpha - \left[\frac{\sqrt{3}}{2} \cdot \operatorname{sen}\alpha - \frac{1}{2} \cdot \cos\alpha\right] =$$

$$= -\cancel{\frac{1}{2}\cos\alpha} + \cancel{\frac{\sqrt{3}}{2}\operatorname{sen}\alpha} + \cancel{\frac{1}{2}\operatorname{sen}\alpha} - \frac{\sqrt{3}}{2}\cos\alpha - \cancel{\frac{1}{2}\operatorname{sen}\alpha} -$$

$$- \cancel{\frac{\sqrt{3}}{2}\operatorname{sen}\alpha} + \cancel{\frac{1}{2}\cos\alpha} = -\frac{\sqrt{3}}{2}\cos\alpha$$

OK.

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$$\cos\left(\frac{3}{4}\pi+2\alpha\right)-\cos\left(2\alpha+\frac{\pi}{4}\right)+\sqrt{2}\cos^2\alpha=$$

$$\cos\frac{3}{4}\pi\cdot\cos 2\alpha-\operatorname{sen}\frac{3}{4}\cdot\operatorname{sen}2\alpha-\left[\cos 2\alpha\cdot\cos\frac{\pi}{4}-\operatorname{sen}2\alpha\cdot\operatorname{sen}\frac{\pi}{4}\right]+$$

$$+\sqrt{2}\cos^2\alpha=$$

$$=-\frac{\sqrt{2}}{2}\cdot\cos 2\alpha-\frac{\sqrt{2}}{2}\cdot\operatorname{sen}2\alpha-\left[\frac{\sqrt{2}}{2}\cdot\cos 2\alpha-\frac{\sqrt{2}}{2}\operatorname{sen}2\alpha\right]+\sqrt{2}\cos^2\alpha=$$

$$=\underline{-\frac{\sqrt{2}}{2}\cdot\cos 2\alpha}\cancel{-\frac{\sqrt{2}}{2}\cdot\operatorname{sen}2\alpha}\underline{-\frac{\sqrt{2}}{2}\cos 2\alpha}\cancel{+\frac{\sqrt{2}}{2}\operatorname{sen}2\alpha}+\sqrt{2}\cos^2\alpha=$$

$$=-\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}}\cos 2\alpha+\sqrt{2}\cos^2\alpha=-\sqrt{2}\cos 2\alpha+\sqrt{2}\cos^2\alpha=$$

$$=-\sqrt{2}\left(2\cos^2\alpha-1\right)+\sqrt{2}\cos^2\alpha=-2\sqrt{2}\cos^2\alpha+\sqrt{2}+\sqrt{2}\cos^2\alpha=$$

$$=\sqrt{2}-\sqrt{2}\cos^2\alpha=\sqrt{2}-\sqrt{2}\left(1-\operatorname{sen}^2\alpha\right)=\cancel{\sqrt{2}}-\cancel{\sqrt{2}}+\sqrt{2}\operatorname{sen}^2\alpha=$$

$$=\sqrt{2}\operatorname{sen}^2\alpha$$

OK

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$$\frac{tg\left(\frac{\pi}{4}+\alpha\right)\cdot tg\left(\frac{\pi}{4}-\alpha\right)-1}{tg\left(\alpha+\frac{\pi}{4}\right)+tg\left(\frac{3}{4}\pi+\alpha\right)} =$$

$$= \frac{\frac{tg\frac{\pi}{4}+tg\,\alpha}{1-tg\frac{\pi}{4}\cdot tg\,\alpha}\cdot\frac{tg\frac{\pi}{4}-tg\,\alpha}{1+tg\frac{\pi}{4}\cdot tg\,\alpha}-1}{\frac{tg\,\alpha+tg\frac{\pi}{4}}{1-tg\,\alpha\cdot tg\frac{\pi}{4}}+\frac{tg\frac{3}{4}\pi+tg\,\alpha}{1-tg\frac{3}{4}\pi\cdot tg\,\alpha}} =$$

$$= \frac{\frac{1+\cancel{tg\,\alpha}}{1-\cancel{tg\,\alpha}}\cdot\frac{1-\cancel{tg\,\alpha}}{1+\cancel{tg\,\alpha}}-1}{\frac{tg\,\alpha+1}{1-tg\,\alpha}+\frac{-1+tg\,\alpha}{1+tg\,\alpha}} =$$

$$= \frac{0}{\dfrac{(1+tg\alpha)^2+(tg\alpha-1)(1-tg\alpha)}{(1-tg\alpha)(1+tg\alpha)}} = 0$$

OK

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$$2\cos\left(\alpha-\frac{\beta}{2}\right)\cdot \operatorname{sen}\frac{\beta}{2} - 2\cos^2\frac{\alpha}{2}\operatorname{sen}\beta + \operatorname{sen}\alpha\cos\beta =$$

$$=2\left[\cos\alpha\cdot\cos\frac{\beta}{2}+\operatorname{sen}\alpha\operatorname{sen}\frac{\beta}{2}\right]\cdot\operatorname{sen}\frac{\beta}{2} -$$

$$-\cancel{2}\cdot\frac{1+\cos\alpha}{\cancel{2}}\cdot\operatorname{sen}\beta+\operatorname{sen}\alpha\cos\beta=$$

$$=2\left[\cos\alpha\cdot\cos\frac{\beta}{2}\cdot\operatorname{sen}\frac{\beta}{2}+\operatorname{sen}\alpha\cdot\operatorname{sen}\frac{\beta}{2}\cdot\operatorname{sen}\frac{\beta}{2}\right]-$$

$$-(1+\cos\alpha)\cdot\operatorname{sen}\beta+\operatorname{sen}\alpha\cos\beta=$$

$$=2\cos\alpha\cos\frac{\beta}{2}\operatorname{sen}\frac{\beta}{2}+2\operatorname{sen}\alpha\operatorname{sen}\frac{\beta}{2}\cdot\operatorname{sen}\frac{\beta}{2}-$$

$$-\operatorname{sen}\beta-\cos\alpha\operatorname{sen}\beta+\operatorname{sen}\alpha\cos\beta=$$

$$=2\cos\alpha\cos\frac{\beta}{2}\operatorname{sen}\frac{\beta}{2}+2\operatorname{sen}\alpha\cdot\operatorname{sen}^2\frac{\beta}{2}-\operatorname{sen}\beta-\cos\alpha\operatorname{sen}\beta+$$

$$+\operatorname{sen}\alpha\cos\beta=$$

Noi sappiamo che

$$\operatorname{sen}\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}=\frac{1}{2}\operatorname{sen}\alpha$$

$$=\cancel{2}\cos\alpha\left(\frac{1}{\cancel{2}}\operatorname{sen}\beta\right)+\cancel{2}\operatorname{sen}\alpha\left(\frac{1-\cos\beta}{\cancel{2}}\right)-\operatorname{sen}\beta-\cos\alpha\operatorname{sen}\beta+$$

$$+\operatorname{sen}\alpha\cos\beta=$$

$$=\cancel{\cos\alpha\operatorname{sen}\beta}+\operatorname{sen}\alpha-\operatorname{sen}\alpha\cos\beta-\operatorname{sen}\beta-\cancel{\cos\alpha\operatorname{sen}\beta}+$$

$$+\operatorname{sen}\alpha\cos\beta=$$

$$=\operatorname{sen}\alpha-\cancel{\operatorname{sen}\alpha\cos\beta}-\operatorname{sen}\beta+\cancel{\operatorname{sen}\alpha\cos\beta}=$$

$$=\operatorname{sen}\alpha-\operatorname{sen}\beta$$

OK

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$$\left[\cos^2\left(\frac{\alpha-\beta}{2}\right)-\operatorname{sen}^2\left(\frac{\alpha+\beta}{2}\right)\right]\cdot\frac{\operatorname{sen}(\pi+\beta)}{\operatorname{sen}2\alpha\cdot\operatorname{sen}2\beta}=$$

$$=\left[\cos\left(\frac{\alpha-\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)-\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\cdot\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\right]\cdot$$

$$\cdot\frac{\overset{0}{\operatorname{sen}\pi}\cos\beta+\overset{(-1)}{\cos\pi}\cdot\operatorname{sen}\beta}{2\operatorname{sen}\alpha\cos\alpha\cdot2\operatorname{sen}\beta\cos\beta}=$$

Applichiamo le formule di Werner adattate opportunamente

$$=\left[\frac{1}{2}\left[\cos\left(\frac{\alpha-\beta}{2}+\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}-\frac{\alpha-\beta}{2}\right)\right]-\right.$$

(12)

$$-\frac{1}{2}\left[\cos\left(\frac{\alpha+\beta}{2}-\frac{\alpha+\beta}{2}\right)-\cos\left(\frac{\alpha+\beta}{2}+\frac{\alpha+\beta}{2}\right)\right]\Bigg]\cdot$$

$$\cdot\frac{-\cancel{\operatorname{sen}\beta}}{4\operatorname{sen}\alpha\cos\alpha\cancel{\operatorname{sen}\beta}\cos\beta}=$$

$$=\frac{1}{2}\left[\cos\left(\frac{\alpha-\beta+\alpha-\beta}{2}\right)+\cos\left(\frac{\cancel{\alpha}-\beta-\cancel{\alpha}+\beta}{2}\right)\right]-$$

$$-\frac{1}{2}\left[\cos\left(\frac{\cancel{\alpha}+\cancel{\beta}-\cancel{\alpha}-\cancel{\beta}}{2}\right)-\cos\left(\frac{\alpha+\beta+\alpha+\beta}{2}\right)\right]\cdot\frac{-1}{4\operatorname{sen}\alpha\cos\alpha\cos\beta}=$$

$$=\frac{1}{2}\left[\cos\left(\frac{2\alpha-2\beta}{2}\right)+\cos 0^\circ\right]-\frac{1}{2}\left[\cos 0^\circ-\cos\left(\frac{2\alpha+2\beta}{2}\right)\right]\cdot$$

$$\cdot \; \frac{-1}{4\,\text{sen}\,\alpha \cos\alpha \cos\beta} =$$

$$= \frac{1}{2}\left[\cos\left(\frac{\cancel{2}(\alpha-\beta)}{\cancel{2}}\right) + 1\right] - \frac{1}{2}\left[1 - \cos\left(\frac{\cancel{2}(\alpha+\beta)}{\cancel{2}}\right)\right] \cdot$$

$$\cdot \; \frac{-1}{4\,\text{sen}\,\alpha \cos\alpha \cos\beta} =$$

$$= \frac{1}{2}\left[\cos(\alpha-\beta) + 1\right] - \frac{1}{2}\left[1 - \cos(\alpha+\beta)\right] \cdot \frac{-1}{4\,\text{sen}\,\alpha \cos\alpha \cos\beta} =$$

$$= \left[\frac{1}{2}\cos(\alpha-\beta) + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \frac{1}{2}\cos(\alpha+\beta)\right] \cdot \frac{-1}{4\,\text{sen}\,\alpha \cos\alpha \cos\beta} =$$

$$= \left[\frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)\right] \cdot \frac{-1}{4\,\text{sen}\,\alpha\cos\alpha\cos\beta} =$$

$$= \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right] \cdot \frac{-1}{4\,\text{sen}\,\alpha\cos\alpha\cos\beta} =$$

$$= \frac{1}{2}\left[\cos\alpha\cos\beta + \cancel{\text{sen}\,\alpha\,\text{sen}\,\beta} + \cos\alpha\cos\beta - \cancel{\text{sen}\,\alpha\,\text{sen}\,\beta}\right] \cdot$$

$$\cdot \frac{-1}{4\,\text{sen}\,\alpha\cos\alpha\cos\beta} = \frac{1}{\cancel{2}}\left(\cancel{2}\cos\alpha\cancel{\cos\beta}\right) \cdot \frac{-1}{4\,\text{sen}\,\alpha\,\cancel{\cos\alpha}\,\cancel{\cos\beta}} =$$

$$= -\frac{1}{4\,\text{sen}\,\alpha}$$

:)

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$$\frac{(\operatorname{sen}\alpha+\cos\alpha)^2-\operatorname{sen}2\alpha}{(\operatorname{cosec}\alpha+\operatorname{cotg}\alpha)^2}-\frac{1-\cos\alpha}{1+\cos\alpha}=$$

$$=\frac{\operatorname{sen}^2\alpha+\cos^2\alpha+2\operatorname{sen}\alpha\cos\alpha-2\operatorname{sen}\alpha\cos\alpha}{\left(\frac{1}{\operatorname{sen}\alpha}+\frac{\cos\alpha}{\operatorname{sen}\alpha}\right)^2}-\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$=\frac{1+0}{\left(\frac{1+\cos\alpha}{\operatorname{sen}\alpha}\right)^2}-\frac{1-\cos\alpha}{1+\cos\alpha}=$$

$$=\frac{1}{\frac{1+\cos^2\alpha+2\cos\alpha}{\operatorname{sen}^2\alpha}}-\frac{1-\cos\alpha}{1+\cos\alpha}=$$

$$=\frac{\operatorname{sen}^2\alpha}{1+\cos^2\alpha+2\cos\alpha}-\frac{1-\cos\alpha}{1+\cos\alpha}=$$

$$=\frac{\operatorname{sen}^2\alpha}{(1+\cos\alpha)^2}-\frac{1-\cos\alpha}{1+\cos\alpha}=$$

$$= \frac{\operatorname{sen}^2\alpha - (1-\cos\alpha)(1+\cos\alpha)}{(1+\cos\alpha)^2} =$$

$$= \frac{\operatorname{sen}^2\alpha - (1-\cos^2\alpha)}{(1+\cos\alpha)^2} = \frac{\operatorname{sen}^2\alpha - 1 + \cos^2\alpha}{(1+\cos\alpha)^2} =$$

$$= \frac{1-1}{(1+\cos\alpha)^2} = 0$$

OK

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$$\frac{\cos 3\alpha + \cos 4\alpha + \cos 5\alpha}{\operatorname{sen} 3\alpha + \operatorname{sen} 4\alpha + \operatorname{sen} 5\alpha} =$$

applichiamo le formule di prostaferesi a 5α e 3α

$$= \frac{2\cos\frac{3\alpha+5\alpha}{2}\cdot\cos\frac{5\alpha-3\alpha}{2} + \cos 4\alpha}{2\operatorname{sen}\frac{3\alpha+5\alpha}{2}\cdot\cos\frac{5\alpha-3\alpha}{2} + \operatorname{sen} 4\alpha} =$$

$$= \frac{2\cos 4\alpha\cdot\cos\alpha + \cos 4\alpha}{2\operatorname{sen} 4\alpha\cdot\cos\alpha + \operatorname{sen} 4\alpha} =$$

$$= \frac{\cos 4\alpha\,(\cancel{2\cos\alpha + 1})}{\operatorname{sen} 4\alpha\,(\cancel{2\cos\alpha + 1})} = \cot g\, 4\alpha$$

OK

Esercizi presi dal libro: (18)

G. Zwirner, L. Scaglianti

Funzioni in $\mathbb{R}$

1A

Casa editrice: CEDAM

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$$\frac{\left(\operatorname{sen}\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2-1}{\operatorname{tg}\frac{\alpha}{2}-\operatorname{sen}\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}=2\operatorname{ctg}^2\frac{\alpha}{2}$$

"Ricordiamo che dalle formule di duplicazione

$\operatorname{sen}2\alpha=2\operatorname{sen}\alpha\cos\alpha$. Sostituiamo α con $\frac{\alpha}{2}$:

$\operatorname{sen}\cancel{2}\cdot\frac{\alpha}{\cancel{2}}=2\operatorname{sen}\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$; $\boxed{\operatorname{sen}\alpha=2\operatorname{sen}\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}$ inoltre

$\boxed{\frac{1}{2}\operatorname{sen}\alpha=\operatorname{sen}\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}$ "

$$\frac{\overbrace{\operatorname{sen}^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}^{\to 1}+2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}-1}{\operatorname{tg}\frac{\alpha}{2}-\frac{1}{2}\operatorname{sen}\alpha}=2\operatorname{ctg}^2\frac{\alpha}{2}\ ;$$

$$\frac{1-1+2\cdot\frac{1}{2}\operatorname{sen}\alpha}{\operatorname{tg}\frac{\alpha}{2}-\frac{1}{2}\operatorname{sen}\alpha}=2\operatorname{ctg}^2\frac{\alpha}{2}\ ;$$

$$\frac{\operatorname{sen}\alpha}{tg\frac{\alpha}{2}-\frac{1}{2}\operatorname{sen}\alpha}=2\,ctg^2\frac{\alpha}{2}$$

$$\frac{\operatorname{sen}\alpha}{\frac{2tg\frac{\alpha}{2}-\operatorname{sen}\alpha}{2}}=2\,ctg^2\frac{\alpha}{2}\ ;$$

$$\operatorname{sen}\alpha\cdot\frac{2}{2tg\frac{\alpha}{2}-\operatorname{sen}\alpha}=2\,ctg^2\frac{\alpha}{2}$$

$$2\operatorname{sen}\alpha=\left(2\,ctg^2\frac{\alpha}{2}\right)\left(2tg\frac{\alpha}{2}-\operatorname{sen}\alpha\right)$$

$$2 \operatorname{sen}\alpha = 2\left(\frac{1+\cos\alpha}{\operatorname{sen}\alpha}\right)^2\left(2\left(\frac{\operatorname{sen}\alpha}{1+\cos\alpha}\right) - \operatorname{sen}\alpha\right)$$

$$2 \operatorname{sen}\alpha = 2 \, \frac{(1+\cos\alpha)^2}{\operatorname{sen}^2\alpha} \cdot \left(\frac{2\operatorname{sen}\alpha}{1+\cos\alpha} - \operatorname{sen}\alpha\right);$$

$$2 \operatorname{sen}\alpha = 2 \cdot \frac{(1+\cos\alpha)^{\cancel{2}}}{\operatorname{sen}^{\cancel{2}}\alpha} \cdot \frac{2\,\cancel{\operatorname{sen}\alpha}}{\cancel{(1+\cos\alpha)}} - 2\,\frac{(1+\cos\alpha)^2}{\operatorname{sen}^{\cancel{2}}\alpha} \cdot \cancel{\operatorname{sen}\alpha}$$

$$\cancel{2} \operatorname{sen}\alpha = 2 \cdot \frac{1+\cos\alpha}{\operatorname{sen}\alpha} \cdot \cancel{2} - \cancel{2}\,\frac{(1+\cos\alpha)^2}{\operatorname{sen}\alpha}$$

$$\operatorname{sen}\alpha = \frac{2(1+\cos\alpha)}{\operatorname{sen}\alpha} - \frac{(1+\cos\alpha)^2}{\operatorname{sen}\alpha}$$

$$\operatorname{sen}\alpha - \frac{2(1+\cos\alpha)}{\operatorname{sen}\alpha} + \frac{(1+\cos\alpha)^2}{\operatorname{sen}\alpha} = 0$$

$$\frac{\operatorname{sen}^2\alpha - 2(1+\cos\alpha) + (1+\cos\alpha)^2}{\operatorname{sen}\alpha} = 0$$

$$\operatorname{sen}^2\alpha - 2(1+\cos\alpha) + (1+\cos\alpha)^2 = 0$$

$$\operatorname{sen}^2\alpha - 2 - \cancel{2\cos\alpha} + 1 + \cos^2\alpha + \cancel{2\cos\alpha} = 0$$

$$\operatorname{sen}^2\alpha + \cos^2\alpha - 2 + 1 = 0$$

$$1 - 2 + 1 = 0 \quad ; \quad 0 = 0$$

:)

N170 pag 596 - Zwirner

$$2\,\text{sen}\,\alpha\;\frac{(1-\cos\alpha)}{\text{sen}^2\frac{\alpha}{2}} = \frac{4\,\text{sen}\,\alpha + 2\,\text{sen}\,2\alpha}{1+\cos\alpha}$$

$$2\,\text{sen}\,\alpha \cdot \frac{1-\cos\alpha}{\frac{1-\cos\alpha}{2}} = \frac{4\,\text{sen}\,\alpha + 2(2\,\text{sen}\,\alpha\cos\alpha)}{1+\cos\alpha}$$

$$2\,\text{sen}\,\alpha \cdot (1-\cancel{\cos\alpha}) \cdot \frac{2}{(1-\cancel{\cos\alpha})} = \frac{4\,\text{sen}\,\alpha + 4\,\text{sen}\,\alpha\cos\alpha}{1+\cos\alpha}$$

$$4\,\text{sen}\,\alpha = \frac{4\,\text{sen}\,\alpha + 4\,\text{sen}\,\alpha\cos\alpha}{1+\cos\alpha} \;;$$

$$4 = \frac{4+4\cos\alpha}{1+\cos\alpha} \;;$$

$$\cancel{4}(1+\cos\alpha) = \cancel{4}(1+\cos\alpha) \;;$$

$$1+\cos\alpha = 1+\cos\alpha \implies 0=0 \quad :)$$