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Esercizi svolti sulle equazioni esponenziali

Equazione esponenziale

$$5 \cdot 3^{x-1} - 3 \cdot \frac{5^{2x-1}}{5^{x}} = 0$$

$$5 \cdot 3^{x-1} - 3 \cdot 5^{2x-1} \cdot 5^{-x} = 0, \quad 5 \cdot 3^{x-1} - 3 \cdot 5^{2x-1-x} = 0$$

$$5 \cdot 3^{x-1} - 3 \cdot 5^{x-1} = 0, \quad 5 \cdot 3^{x-1} = 3 \cdot 5^{x-1},$$

applichiamo $\lg_3$ ad ambo i membri.

$$\lg_3 (5 \cdot 3^{x-1}) = \lg_3 (3 \cdot 5^{x-1}),$$

$$\lg_3 5 + \lg_3 3^{x-1} = \cancel{\lg_3}\, 3 + \lg_3 5^{x-1},$$

$$\lg_3 3^{x-1} - \lg_3 5^{x-1} = 1 - \lg_3 5$$

$$(x-1) \cancel{\lg_3} 3 - (x-1) \lg_3 5 = 1 - \lg_3 5$$

$$x - 1 - x \lg_3 5 + \lg_3 5 = 1 - \lg_3 5$$

$$x(1 - \lg_3 5) = 1 - \lg_3 5 + 1 - \lg_3 5;$$

$$x(1 - \lg_3 5) = 2 - 2\lg_3 5; \quad x(1 - \lg_3 5) = 2(1 - \lg_3 5);$$

$$x = \frac{2\cancel{(1 - \lg_3 5)}}{\cancel{(1 - \lg_3 5)}}; \quad x = 2$$

$$\left[\left(\frac{2}{3}\right)^{x}\right]^{2} = \frac{8}{27}$$

$$\left[\left(\frac{2}{3}\right)^{x}\right]^{2} = \frac{2^3}{3^3} \; ; \; \left[\left(\frac{2}{3}\right)^{x}\right]^{2} = \left(\frac{2}{3}\right)^{3} \; ; \; \left(\frac{2}{3}\right)^{2x} = \left(\frac{2}{3}\right)^{3}$$

$$\cancel{\log_{\frac{2}{3}}\left(\frac{2}{3}\right)}^{2x} = \cancel{\log_{\frac{2}{3}}\left(\frac{2}{3}\right)}^{3} \; ; \; 2x = 3 \; ; \; x = \frac{3}{2}$$

$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{3}{2}\right)^{2x} \; ; \; \cancel{\log_{\frac{2}{3}}\left(\frac{2}{3}\right)}^{x+1} = \log_{\frac{2}{3}}\left(\frac{3}{2}\right)^{2x} \; ;$$

$$x+1 = 2x \cdot \log_{\frac{2}{3}} \frac{3}{2} \; ; \; x+1 = 2x \cdot \left(-\log_{\frac{3}{2}} \frac{3}{2}\right) \; ;$$

$$x+1 = 2x \cdot (-1) \; ; \; x+1 = -2x \; ; \; x+2x = -1 \; ; \; 3x = -1 \; ;$$

$$x = -\frac{1}{3}$$

$$\frac{3^{1-x}\cdot 9^{2+x}}{27^{x}} = \frac{1}{3}$$

$$\frac{3\cdot 3^{-x}\cdot \left(3^{2}\right)^{2+x}}{\left(3^{3}\right)^{x}} = 3^{-1} \ ; \quad \frac{3\cdot 3^{-x}\cdot 3^{4+2x}}{3^{3x}} = 3^{-1} \ ;$$

$$3^{1-x+4+2x}\cdot 3^{3x} = 3^{-1} \ ; \quad 3^{5+x}\cdot 3^{3x} = 3^{-1} \ ;$$

$$3^{5+x-3x} = 3^{-1} \ ; \quad 3^{5-2x} = 3^{-1} \ ;$$

$$\cancel{\log_3} 3^{5-2x} = \cancel{\log_3} 3^{-1} \ ; \quad 5-2x=-1 \ ; \quad -2x=-5-1 \ ;$$

$$-2x=-6 \ ; \quad x=3$$

$$\frac{\sqrt[3]{32^{x}}}{\left(2^{x+2}\right)^{x-2}} = 1$$

$$\frac{\sqrt[3]{\left(2^{5}\right)^{x}}}{2^{(x+2)(x-2)}} = 1 \;;\; \sqrt[3]{2^{5x}} = 2^{(x+2)(x-2)}$$

$$\left(2^{5x}\right)^{\frac{1}{3}} = 2^{(x+2)(x-2)} \;;\; 2^{\frac{5}{3}x} = 2^{(x+2)(x-2)}$$

$$\log_2 2^{\frac{5}{3}x} = \log_2 2^{(x+2)(x-2)} \;;\; \frac{5}{3}x = (x+2)(x-2) \;;$$

$$x^2 - 4 = \frac{5}{3}x \;;\; x^2 - \frac{5}{3}x - 4 = 0 \;;\; \frac{3x^2 - 5x - 12}{3} = \frac{0}{3} \;;$$

$$3x^2 - 5x - 12 = 0 \;;\; x = \frac{5 \pm \sqrt{25 - 4(3)(-12)}}{6} =$$

$$= \frac{5 \pm \sqrt{25+144}}{6} = \frac{5 \pm 13}{6} = \begin{cases} \frac{5+13}{6} = \frac{18}{6} = 3 \\ \frac{5-13}{6} = \frac{\cancel{-8}^{4}}{\cancel{6}_{3}} = -\frac{4}{3} \end{cases}$$

$$x_1 = 3 \;;\; x_2 = -\frac{4}{3}$$

$$\frac{32-3^x}{5+3^{-x}} = \frac{9}{2}$$

$$2(32-3^x) = 9(5+3^{-x}) \; ; \; 2(2^5-3^x) = 9\left(5+\frac{1}{3^x}\right)$$

poniamo $3^x = t$

$$2(2^5-t) = 9\left(5+\frac{1}{t}\right) \; ; \; 2^6-2t = 9\left(\frac{5t+1}{t}\right) \; ;$$

$$2^6-2t = \frac{45t+9}{t} \; ; \; t(2^6-2t) = 45t+9 \; ;$$

$$2^6t-2t^2-45t-9=0 \; ; \; -2t^2+t(2^6-45)-9=0$$

$$-2t^2+t(64-45)-9=0 \; ; \; -2t^2+19t-9=0 \; ;$$

$$2t^2-19t+9=0 \; ; \; t = \frac{19 \pm \sqrt{19^2-4(2)(9)}}{4} = \frac{19 \pm \sqrt{361-72}}{4} =$$

$$= \frac{19 \pm 17}{4} = \begin{cases} \frac{36}{4} = 9 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$$

$t_1 = 9$ $\quad$ $t_2 = \frac{1}{2}$

1) $3^x = t_1 \; ; \; 3^x = 9 \; ; \; 3^x = 3^2 \; ; \; x = 2$

2) $3^x = t_2 \; ; \; 3^x = \frac{1}{2} \; ; \; \lg_3 3^x = \lg_3 3^{\frac{1}{2}} \; ; \; x \lg_3 3 = \frac{1}{2} \lg_3 3 \; ;$

$x = \frac{1}{2}$