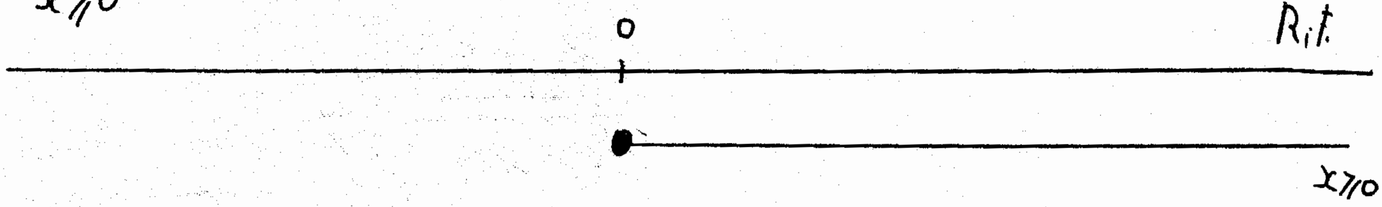
Esercizio - Disequazione Irrazionale Fratta

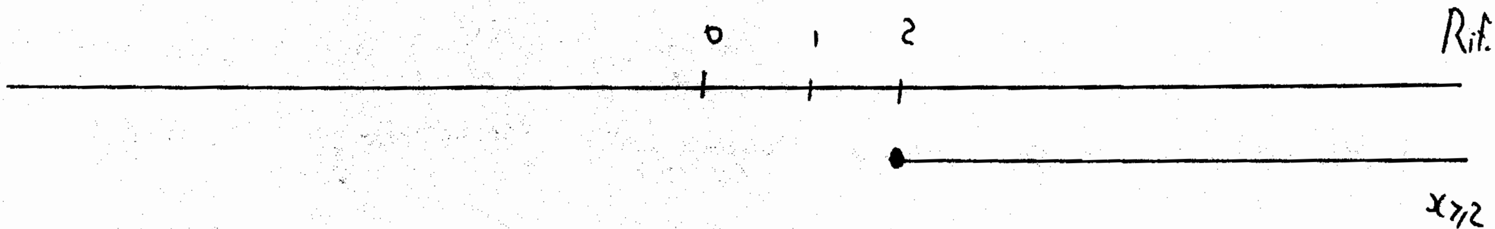
$$\frac{\sqrt{x}+\sqrt{x-2}-1}{\sqrt{x^2+3}-\sqrt{x+5}} \geq 0 \qquad (E)$$

$$\text{C.E.}: \begin{cases} (1) & x \geq 0 \\ (2) & x-2 \geq 0 \\ (3) & x^2+3 \geq 0 \\ (4) & x+5 \geq 0 \\ (5) & \sqrt{x^2+3}-\sqrt{x+5} \neq 0 \end{cases}$$

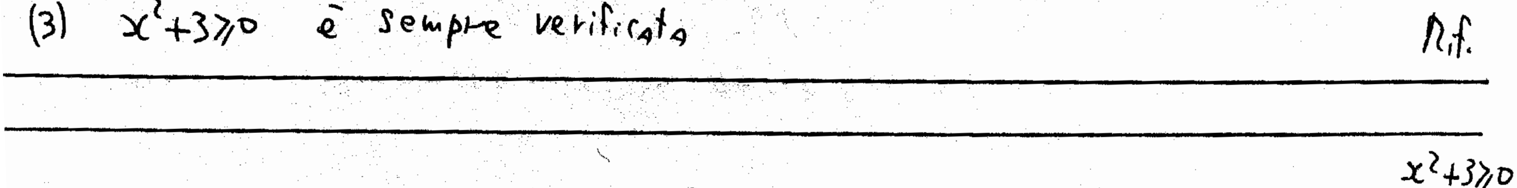
(1) $x \geq 0$



(2) $x \geq 2$

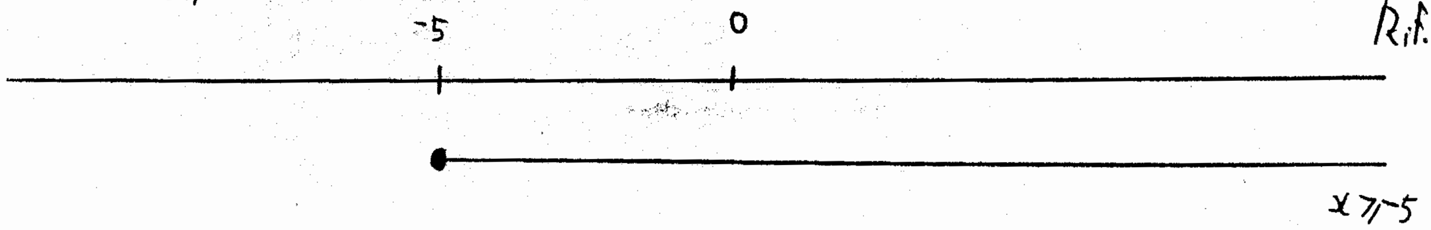


(3) $x^2+3 \geq 0$ è sempre verificata



(4) $x+5 \geq 0$

$x \geq -5$



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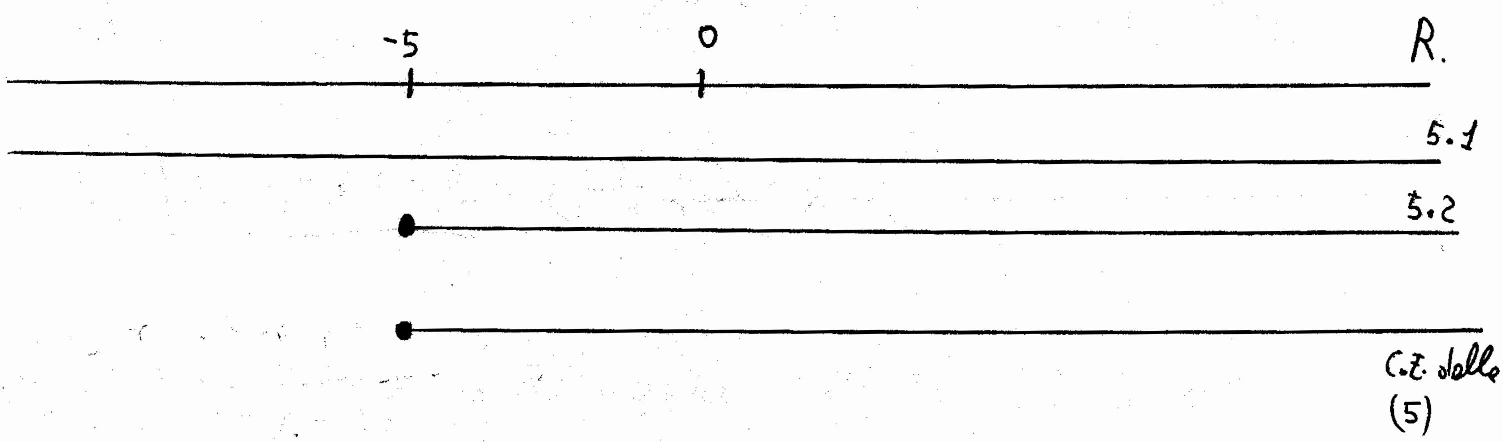
(5) $\sqrt{x^2+3} - \sqrt{x+5} = 0$ e poi calcoliamo il complementare per trovare il $\neq 0$

C.E. della (5): $\begin{cases} x^2+3 \geq 0 & 5.1 \\ x+5 \geq 0 & 5.2 \end{cases}$

(5.1) e (5.2) già studiate

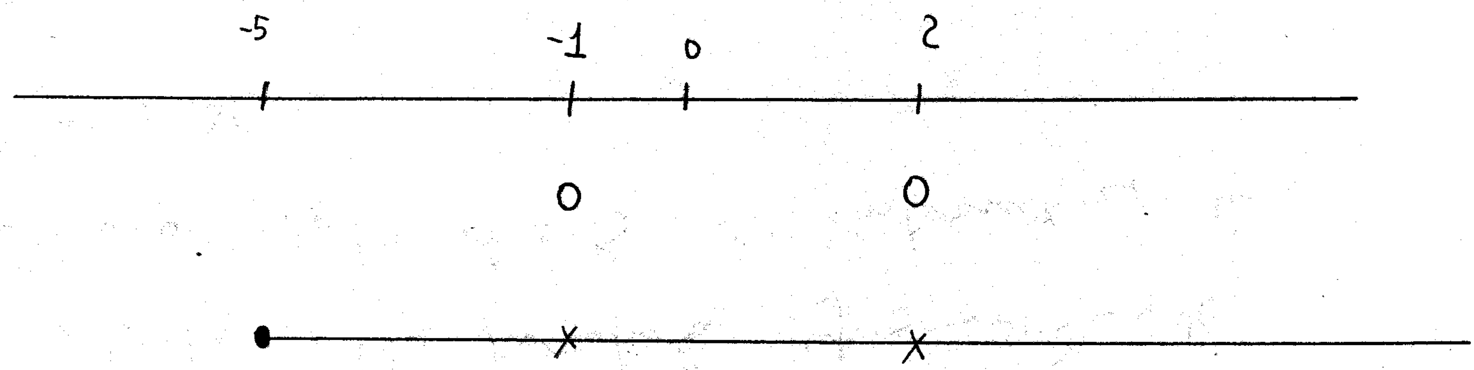


studiamo la (5) e vediamo quali soluzioni appartengono al C.E. della (5)

$$\sqrt{x^2+3} = \sqrt{x+5}$$

$x^2+3=x+5$; $x^2+3-x-5=0$; $x^2-x-2=0$

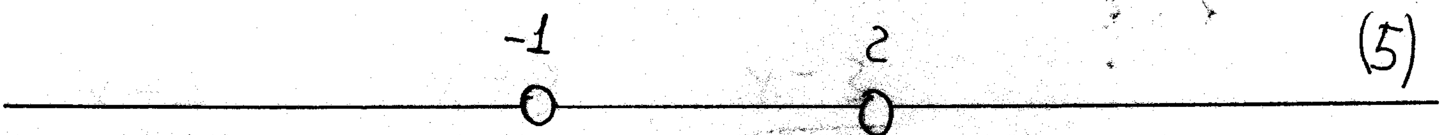
$$x=\frac{1\pm\sqrt{1-4(1)(2)}}{2}=\frac{1\pm 3}{2}= \begin{cases} \dfrac{1+3}{2}=2 \\[2ex] \dfrac{1-3}{2}=-1 \end{cases}$$



Le soluzioni $x=-1$ e $x=2$ $\in$ all'esistenza

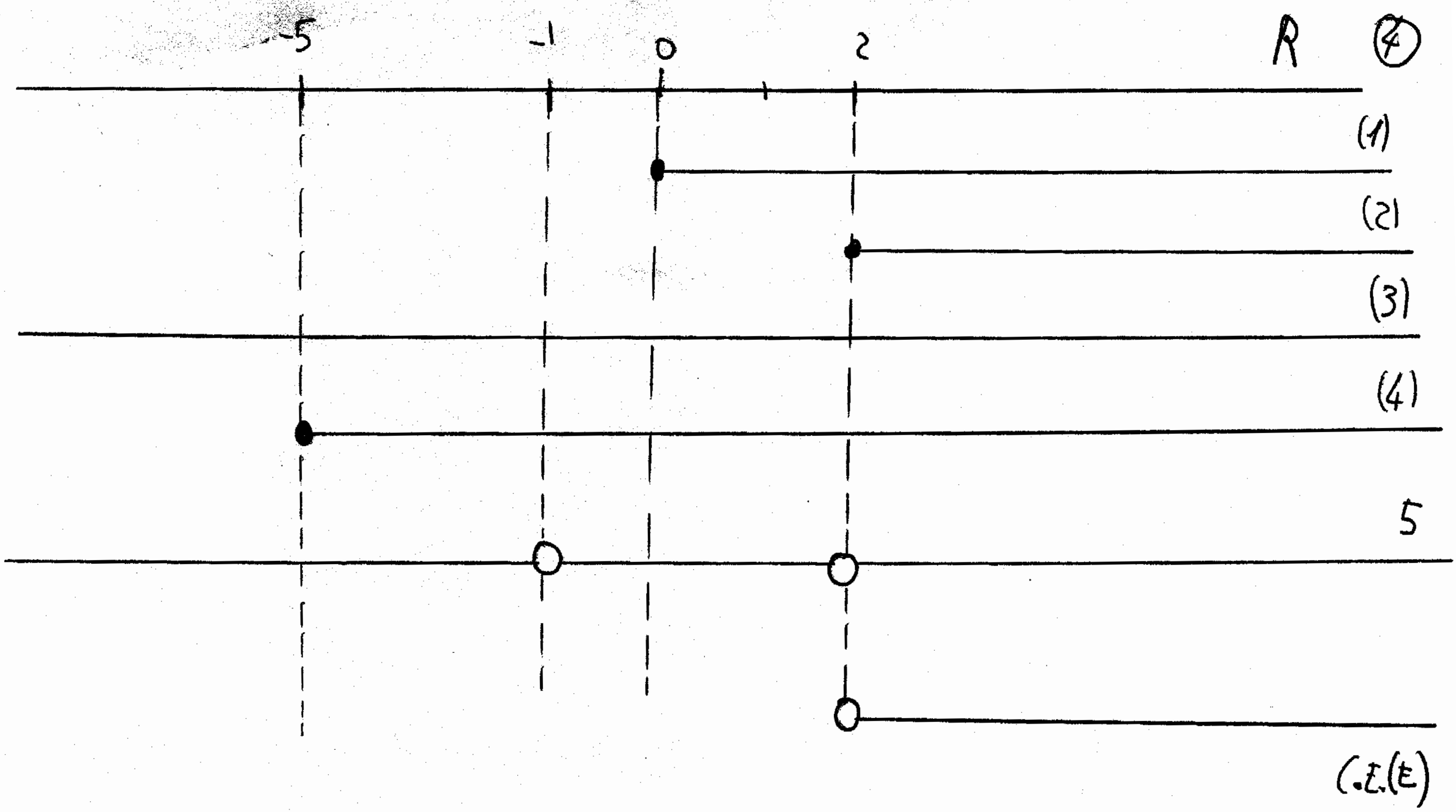
Lo studio del $\neq 0$ di $\sqrt{x^2+3}-\sqrt{x+5}\neq 0$

lo otteniamo mediante il passaggio al complementare



(5)

Studiamo adesso l'intersezione : $(1)\cap(2)\cap(3)\cap(4)\cap(5)$



Il campo di esistenza dell'equazione (E) : $x \in]2;+\infty[$

Studiamo $N(x) \geq 0$

$$\sqrt{x} + \sqrt{x-2} - 1 \geq 0$$

$$\sqrt{x} + \sqrt{x-2} \geq 1$$

$$x + x - 2 + 2\sqrt{x}\sqrt{x-2} \geq 1$$

$$2x - 2 + 2\sqrt{x(x-2)} \geq 1$$

$$2\sqrt{x(x-2)} \geq 1-2x+2$$

$$2\sqrt{x(x-2)} \geq -2x+3$$

$$\sqrt{x(x-2)} \geq \frac{-2x+3}{2}$$

questa appartiene alla così che noi nominato 4ª famiglia

$$\sqrt{A(x)} \geq B(x)$$

che è equivalente a studiare il sistema:

$$\begin{cases} A(x) \geq 0 \\ B(x) \leq 0 \end{cases} \cup \begin{cases} B(x) \geq 0 \\ A(x) \geq [B(x)]^2 \end{cases}$$

cioè

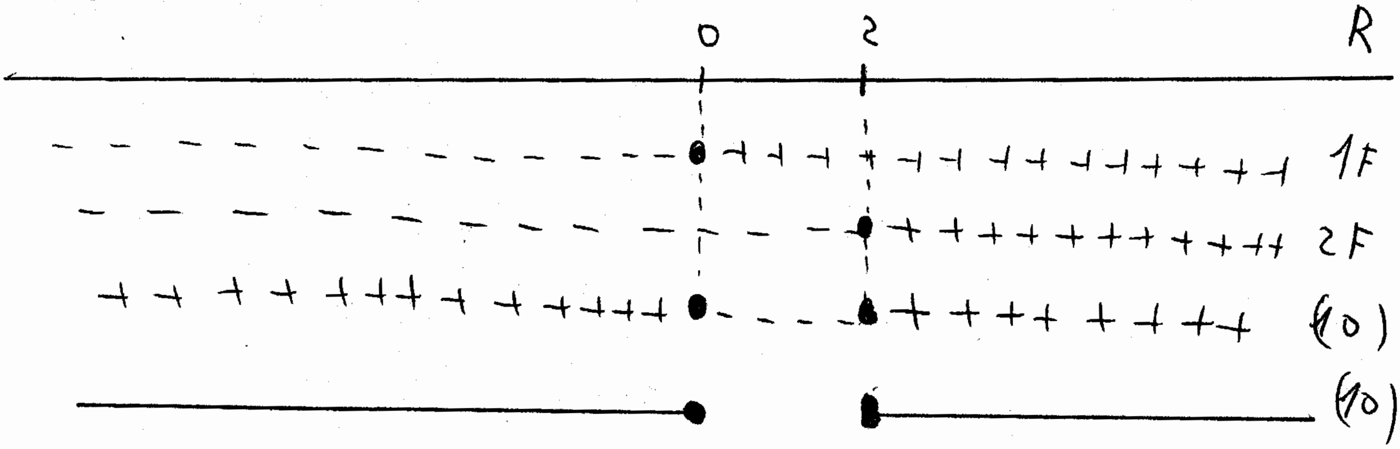
$$\begin{cases} x(x-2) \geq 0 & (10) \\ \dfrac{-2x+3}{2} \leq 0 & (11) \end{cases} \cup \begin{cases} \dfrac{-2x+3}{2} \geq 0 & (12) \\ x(x-2) \geq \left(\dfrac{-2x+3}{2}\right)^2 & (13) \end{cases}$$

(10)

$x(x-2) \geq 0$

1F: $x \geq 0$

2F: $x-2 \geq 0$; $x \geq 2$



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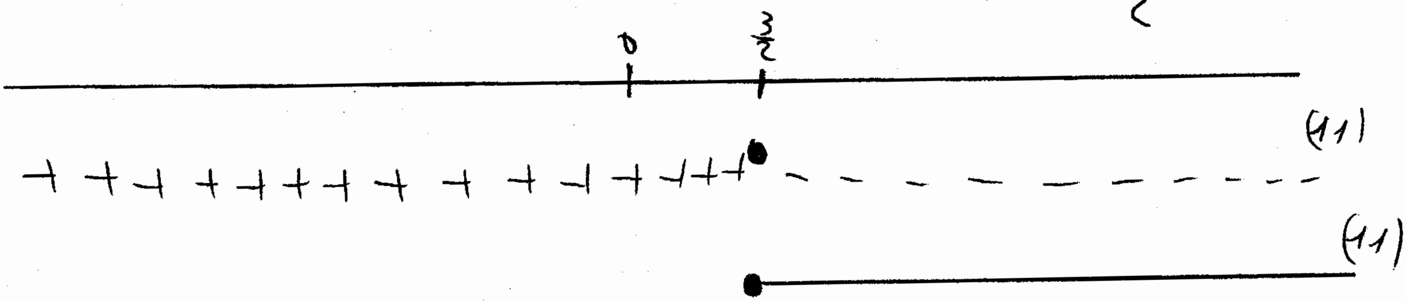
(11)

$N(x) \geq 0$ $\quad -2x+3 \geq 0$; $-2x \geq -3$; $2x \leq 3$; $x \leq \frac{3}{2}$



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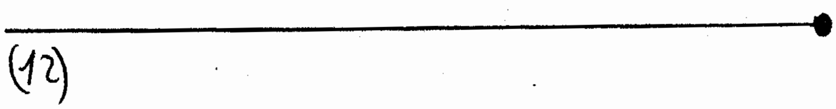
La (12) è immediata:



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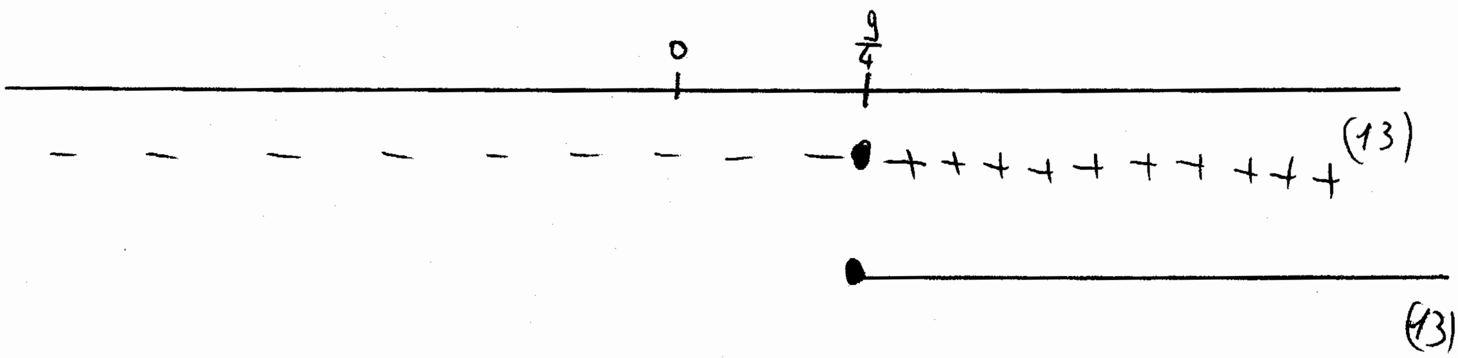
(13)

$$x^2-2x \geqslant \frac{4x^2+9-12x}{4};$$

$$x^2-2x-\frac{4x^2+9-12x}{4}\geqslant 0;\quad \frac{4x^2-8x-(4x^2+9-12x)}{4}\geqslant 0$$

$$\frac{\cancel{4x^2}-8x-\cancel{4x^2}-9+12x}{4}\geqslant 0;\quad \frac{4x-9}{4}\geqslant 0$$

$$4x-9\geqslant 0;\quad 4x\geqslant 9;\quad x\geqslant \frac{9}{4}$$



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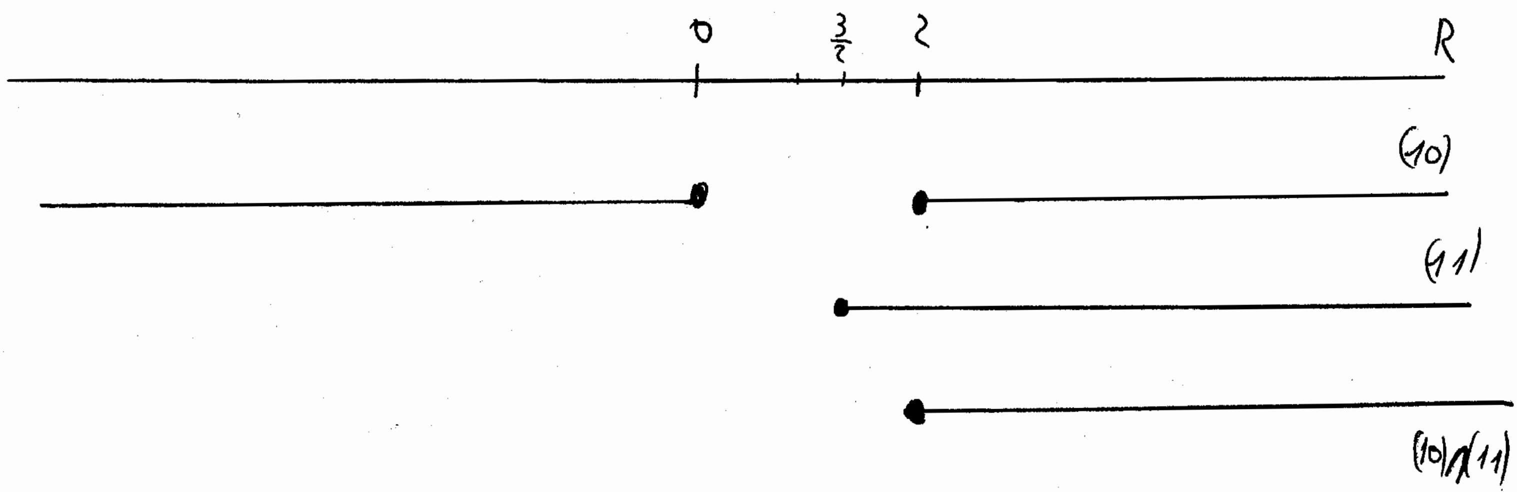
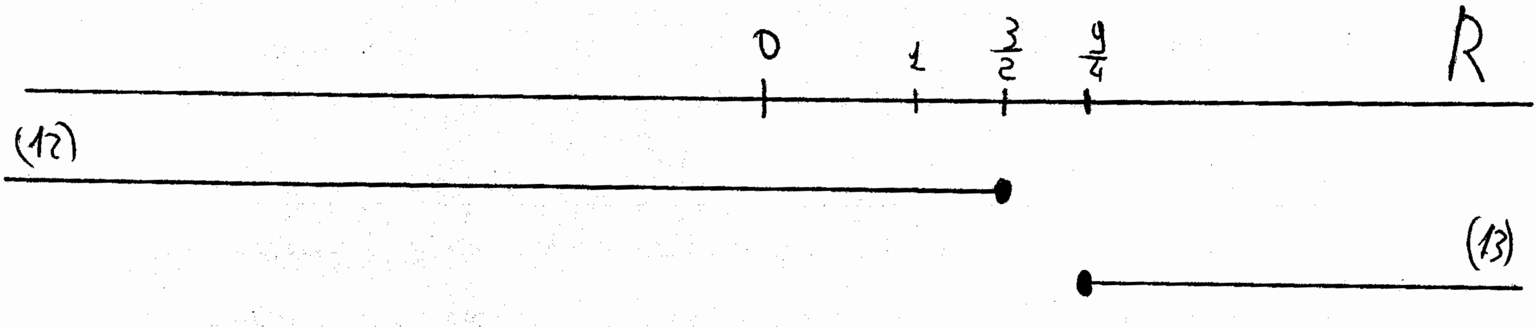
Studiamo $(10)\wedge(11)$



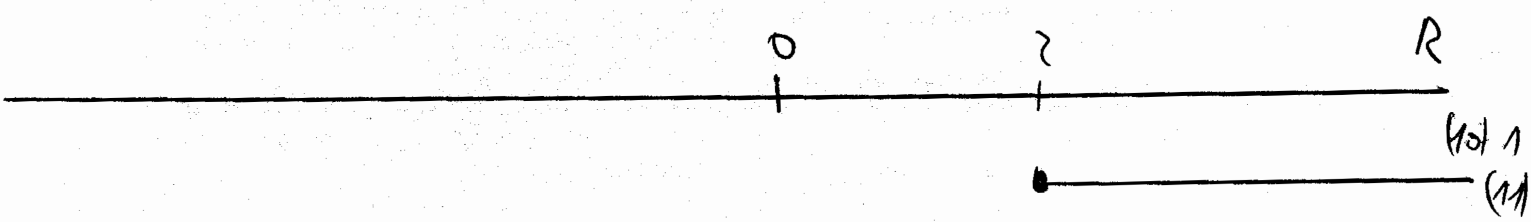
$(12) \land (13)$



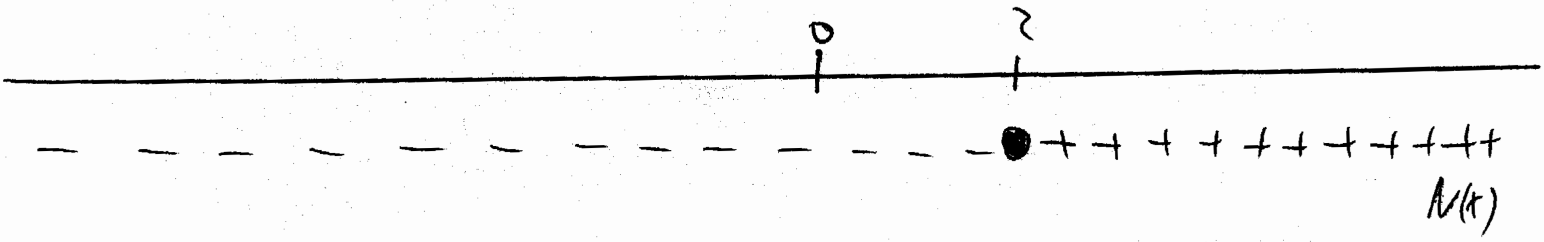
$(12) \land (13) = \emptyset$

Da cui la soluzione dei 2 sistemi è

$(10) \land (11)$, cioè



Visto che questo era lo studio di $N(x) \geq 0$ dobbiamo trasformare la soluzione con i segni espliciti

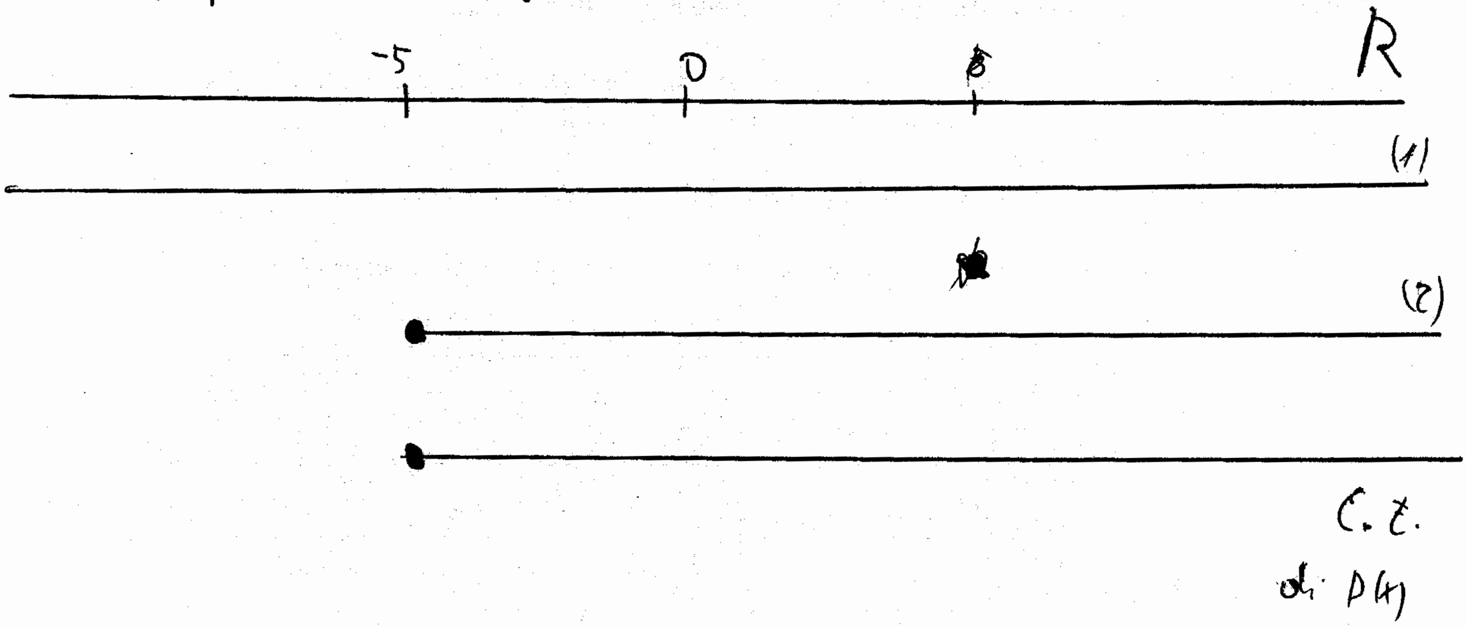


Studiamo $D(x) > 0$

$$\sqrt{x^2+3} - \sqrt{x+5} > 0 \; ;$$

C.E. di $D(x)$ è

$$\begin{cases} x^2+3 \geq 0 & \text{già studiata} \quad (1) \\ x+5 \geq 0 & \text{già studiata} \quad (2) \end{cases}$$



Studiamo l'algebra di $D(x)$: $\sqrt{x^2+3} - \sqrt{x+5} > 0$ (15)

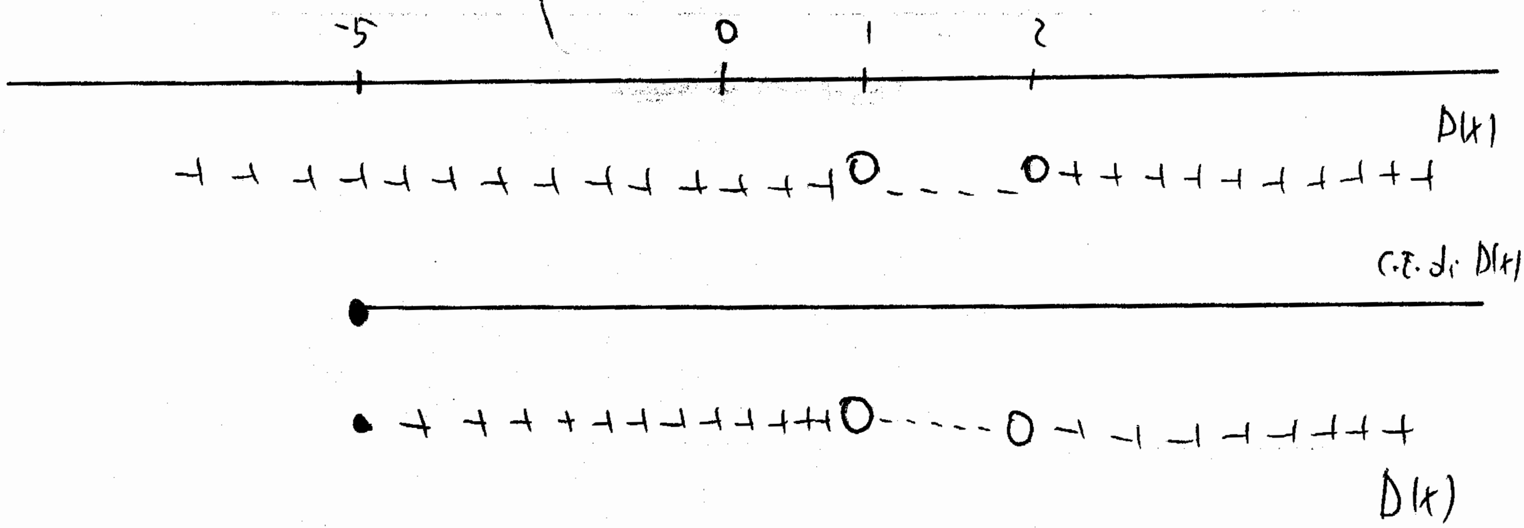
$$\sqrt{x^2+3} > \sqrt{x+5} \; ; \quad x^2+3 > x+5 \; ;$$

$$x^2+3-x-5 > 0 \; ; \quad x^2-x-2 > 0$$

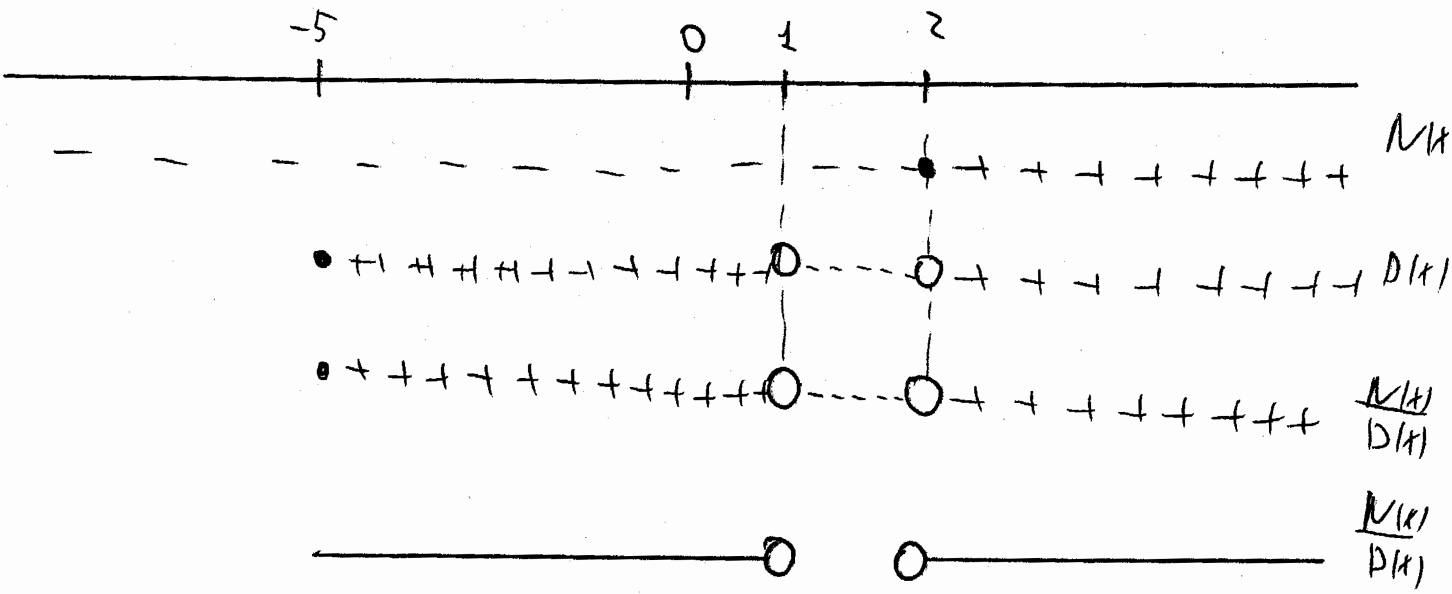
$$\Delta = 1 - 4(1)(-2) = 9$$

$$x = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

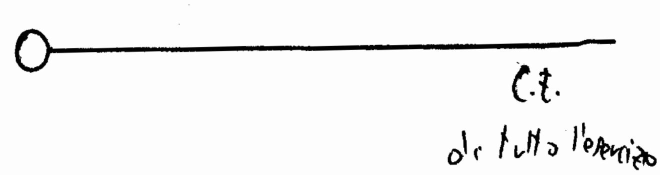
soluzioni esterne

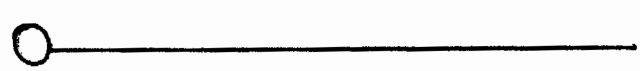


Studiamo $\frac{N(x)}{D(x)}$



Che intersechiamo con il C.E. di tutto l'esercizio





Soluzione Totale

$x > 2$ cioè $]2;+\infty[$:)